MICROFOUNDATIONS OF IS-LM MODELS

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Abstract

The IS-LM model is commonly assumed to be without microfoundations, and therefore not as useful for analytical purposes as microfoundations based models, like the New Keynesian and DSGE models. We show that the IS-LM model is microfoundations based by using the same Lagrangian utility maximization method used in New Keynesian and DSGE models to determine budget constrained utility maximization. Thus, IS-LM models not be dismissed for lack of microfoundations, especially since they appear to have another compelling advantage: there is evidence, which we cite, indicating IS-LM models fit the data better than other models.

1. Microfoundations of IS-LM Models

Use of IS-LM models has been abandoned in recent decades for lack of explicit microfoundations. To many economists, this implies the model has none. Far from lacking microfoundations, we show that IS-LM models are derived from the same microfoundations principles that underly New Keynesian and DSGE economics: utility maximization. This is an important finding since there is evidence that IS-LM fits the data better than other models (Heim2017, Solow 2016).

IS-LM models analyze the determinants of demand for real goods and services (IS curve), the determinants of interest rates (LM curve) and the interrelationship between the two. Microfoundations of such models are assumed nonexistent by some. Other economists would argue microfoundations underly all economic models but sometimes are only implicitly, rather than explicitly expressed (Mankiw 2009).

We also show that aggregate demand (AD) curves can be deduced equally well from the IS_LM model. or the typical microfoundations model, and that the aggregate supply (AS) curves are essentially the same in both models.

1.1. The Utility function

In typical microfoundations models utility is assumed to be derived from consumption (C), (derived from production stemming from willingness to work, which results from consumer demand) and from leisure (L). To illustrate, we move from micro to macro level utility maximization using the following hypothetical utility functions for 3 heterogeneous consumers (assumed for simplicity to represent the whole population)

$U = C^{.9}L^{.1}$; Consumer 1
$U = C^{.5}L^{.5}$ Consumer 2
$U = C^{.1}L^{.9}$ Consumer 3

$\frac{\partial U}{\partial C} = -.9C^{-.1}L^{.9}$; $\frac{\partial U}{\partial L} = .1L^{-.1}C^{.9}$ where C shows declining marginal utility

$\frac{\partial U}{\partial C} = -.5C^{-.5}L^{.5}$ $\frac{\partial U}{\partial L} = .5L^{-.5}C^{.5}$ where L-shows declining marginal utility

$\frac{\partial U}{\partial C} = -.1C^{--.9}L^{.9}$; $\frac{\partial U}{\partial L} = .9L^{-.1}C^{.1}$

For simplicity, assume a price of $1.50, $1.00. or 0.50 for a unit of consumption and $1.00 as the value of a unit of leisure in terms of opportunity cost, lost work income. Total income is assumed to be $100.

Then the Lagrangian (ɛ) utility maximization problem for consumers 1,2 and 3 are

ɛ = $C^{.9}L^{.1}$ –ƛ(($1.50,or $1.00, or $0.50)C + $1.00L - $100) -
ɛ = $C^{.5}L^{.5}$ –ƛ(($1.50, or $1.00, or $0.50)C + $1.00L - $100) -
ɛ = $C^{.1}L^{.9}$ –ƛ(($1.50, or $1.00, or $0.50)C + $1.00L - $100) -

where the loss of leisure is arbitrarily valued at $1.00 per unit of leisure

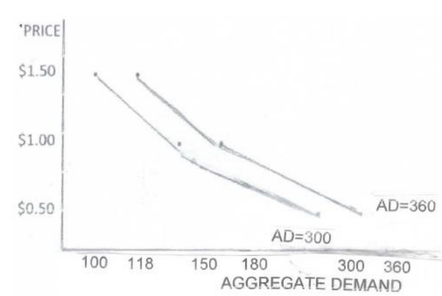
From here it is a simple matter to calculate utility maximizing levels of C growing as the price level declines. This provides the down sloping aggregate demand curve like the one used in IS-LM models Aggregate results are shown in Table 1 below for two income levels to show the rightward shift in aggregate demand expected in IS-LM models when income increases.

Below we show the aggregate demand curve commonly derived from IS-LM modeling, as derived from microfoundations.

Table 1
Units of Consumption Maximizing Utility Given The Price of C

Agg. Income Level = $300.	Agg. Income Level = $360.
Pc = $1.50; C_{1-3}= 100	Pc = $1.50; C_{1-3}= 118
Pc = $1.00; C_{1-3}= 150	Pc = $1.00; C_{1-3}= 180
Pc = $0.50; C_{1-3}= 300	Pc = $0.50; C_{1-3}= 360
AD =	

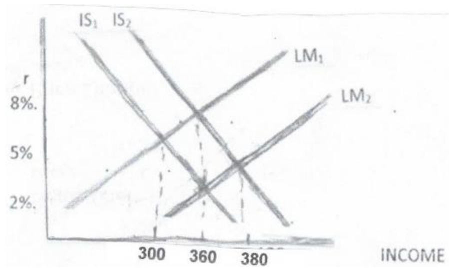
Figure 1
Microfoundations Derived IS-LM Aggregate Demand Curve



The AD curve shifts rightward as income increases. Any increase in the underlying Lagrangian's budget (income) constraint, e.g., from $100 to $120 per individual causes the AD shift. But the reason *why* income increases is unexplained in the Lagrangian model. To find the economics that justifies the increase in the budget constraint we need to look at the economics occurring *before* the Lagrangian process that determines the size of the budget constraint in it. Traditional IS-LM curve economics provides one way of determining the value of income (from the determinants of demand) prior to using the utility maximizing process described by the Lagrangian.

In the IS-LM system, the level of income (Y) is determined by the IS curve, that is by the determinants of C, I, G, X and M, their initial conditions, and their parameters. Once calculated, the economically determined actual value of income (Y) can be obtained using the Lagrangian budget constraint. Equilibrium values of interest rates (r) and income are often calculated simultaneously holding all other IS curve determinants constant. Changes in equilibrium income and interest rates occur when one of the variables held constant changes, shifting either the IS or the LM curve.

Figure 2
Shifts in IS-LM Curves
Income and Interest Rate Effects



Rightward shifts in either the IS or LM curves shift equilibrium income up from a hypothetical 300 to 360. Rightward shifts of both increase it to 380. Interest rates rise with rightward shifts in the IS curve, decrease with rightward shifts of the LM curve. When both curves shift rightward the effect can be to increase or decrease interest rates, depending on the size of the shift in IS compared to LM. A tax cut or an increase in government spending shift the IS curve rightward. An increase in the money supply shifts the LM curve rightward. An increase in inflation shifts both curves leftward; and interest rates rise.

Any IS curve is nothing but the standard national income determination identity

$$Y = C + I + G + X - M$$

In which all the determinants of C, I, G and X-M are held constant except the interest rate. Changes in any of the other determinants of GDP are shown as shifts in the IS curve, not movement along it. We will show below that the national income determination curve can also be deduced from a standard microfoundations model based on a Lagrangian utility optimization function and a budget constraint.

The Upsloping LM curve similarly has microfoundations. It is nothing more than a down sloping money demand curve, and an exogenous (vertical) money supply curve. Interest rates, the price of money, are determined by the intersection of the two curves. The money demand curve shifts rightward as income grows, raising interest rates.

All economic models are microeconomics based, either implicitly or explicitly (Mankiw 2009). In past IS-LM macro models, microfoundations were implicit. The explicit part was the development of the IS-LM curves, and ends with to AD conclusions derived from it. We now construct a microfoundations model which determines the utility maximizing levels of consumption, investment, government goods and services and net exports in any year.

To find the micro foundations of the IS curve, the Lagrangian utility maximization problem ($\mathcal{L}$) to be solved would be (following Clark 2006)

$$\mathcal{L} = C^a I^b G^d NX^e L^f - \lambda(P_c C + P_I I + P_G G + P_{NX} NX + P_L L - \$Income)$$

Where the superscripts are all less than one, and P is the average price for each type of good. As was the case earlier, (L) represents units of leisure. As an example, we will define the utility function to be maximized, subject to a budget constraint, as

$$\mathcal{L} = C^{.4}I^{.2}G^{.1}NX^{.1}L^{.1} - \lambda(\$2.00C + \$100.00I + \$2.00G + \$1.00NX + \$2.00L - \$30{,}000.)$$

Where the marginal utilities of consuming one more unit of any of the commodities in the utility function subject to a budget constraint at the point of utility maximization are

$\frac{\partial U}{\partial C} = .4C^{-.6}I^{.2}\ G^{.1}NX^{.1}L^{.1} - \lambda\$2.00 = 0$; C showing declining marginal utility

$\frac{\partial U}{\partial I} = .2C^{.4}I^{-.8}\ G^{.1}NX^{.1}L^{.1} - \lambda\$100.00 = 0$; I showing declining marginal utility

$\frac{\partial U}{\partial G} = -.1C^{.4}I^{.2}\ G^{-.9}NX^{.1}L^{.1} - \lambda\$2.00 = 0$; G showing declining marginal utility

$\frac{\partial U}{\partial NX} = -.1C^{.4}I^{.2}\ G^{-.1}NX^{-.9}L^{.1} - \lambda\$1.00 = 0$; NX showing declining marginal utility

$\frac{\partial U}{\partial L} = -.1C^{.4}I^{.2}\ G^{-.9}NX^{-.1}L^{-.9} - \lambda\$2.00 = 0$; L showing declining marginal utility. A unit of leisure is arbitrarily valued at \$2.00

$\frac{\partial U}{\partial \lambda} = \$2.00C + \$100.00I + \$2.00G + \$1.00NX + \$2.00L - \$30{,}000. = 0$

Multiplying through the first equation by C/.4, and the second, third, fourth and fifth equations by I/.2, G/.1, NX/.1 and L/.1 and rearranging gives

$$\$2.00C/.4 = \$100.00\ I/.2 = \$2.00\ G/.1 = \$1.00\ NX/.1 = \$2.00L/.1$$

from which income can be shown as equal to

$\$30{,}000 = \$2.00C\ (.4/.4 + .2/.4 + .1/.4 + .1/.4 + .1/.4)$

And the utility maximizing levels of the five commodities as

	Units	Dollar Value
C = .4($30,000)/$2.00(.4+.2+.1+.1+.1) =	6666.7	$ 13333
I = .2($30,000)/$100.00(.4+.2+.1+.1+.1) =	66.67	6667
G = .1($30,000)/$2.00(.4+.2+.1+.1+.1) =	1667.7	3333
NX= .1($30,000)/$1.00(.4+.2+.1+.1+.1) =	3333.3	3333
L = .1($30,000)/$2.00(.4+.2+.1+.1+.1) =	1666.7	3334
		$ 30000 = potential GDP (all Work, no Leisure)
		$ 26666 = actual GDP (mostly Work, some Leisure)

Instead of producing the maximum $30000 worth of goods, with no leisure time, the workers withdraw enough time from maximum possible production ($30000), to enjoy 1667 units of leisure, Lost production is valued at $3334. This decision is the utility maximizing solution to the hypothetical Lagrangian problem shown above.

The IS curve is nothing but a statement of the value of GDP=C+I+G+NX. We have shown the IS curve is derived from standard microfoundations assumptions, like its New Keynesian and DSGE counterparts.

1.2 Supply Curves

In the IS-LM model, profit maximizing aggregate supply curves are often assumed to be flat, either because prices are sticky in the short run, or because markets are competitive. Supply curves can also be upsloping reflecting increased marginal costs due to diminishing returns when holding at least one factor of production constant in the short run. Less than perfectly competitive markets try to manipulate prices and output to maximize profits, shifting the upsloping supply curve up or down. Similarly, if prices are autonomously increased by producers, the IS curve shifts leftward reducing the level of income in the Lagrangian budget constraint. This also shifts the AD curve leftward. Price decreases shift the IS and AD curves the other way.

1.3 Conclusions

The IS-LM system is often assumed to be one without microfoundations, and therefore not as suitable for analytical purposes as more modern microfoundations based models, like the New Keynesian and DSGE models. We have shown that the IS-LM components are as microfoundations based as other macroeconomic models. The IS-LM system components are the aggregates demand and supply curves, and the money demand curves. The aggregate demand curve's microfoundations are developed using the same budget-constrained Lagrangian maximization technique used in New Keynesian and DSGE models.

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