A Dynamic IS-LM Model

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Abstract

I develop a recursive IS-LM model that shows the dynamic path taken when moving from one equilibrium to another. It shows the specific order in which endogenous variable values must be determined. The order in is derived strictly from econometrics. There was only one order in which all the endogenous variables could be solved. The new recursive model did not fit the data as well as traditional static partial equilibrium models of consumption, investment, interest rates, etc. Theoretically, the dynamic model is still to be preferred. Based on accuracy of empirical results, the traditional static IS-LM model seems preferred.

1. The Recursive IS-LM Model

In models of the GDP, parameters indicating how variables are connected to one another are estimated from econometric tests of the variable and its determinants. Some determinants are other endogenous variables, determined by yet other endogenous variables. It is relatively straightforward to estimate the effects on any one endogenous variable of a change in any of its determinants (partial equilibrium modeling). But the change in this variable may recursively affect many of the GDP's other components, whose fee back effects cause further changes in GDP. The process reiterates itself until a new equilibrium GDP is reached.

In models from the 1950 and 1960's, it was common to use simultaneous solution methods to determine multi-equation models. Yet to have a truly scientific system, the system causes must precede their own effect. (Krauss, 2012). Hence, in reality, solving equations determining GDP is a dynamic, recursive process. We often try to proxy that process by using simultaneous equation models to show movement from one equilibrium to another when there is a shock. Everything is determined simultaneously, so al we can show is equilibrium points before and after the shock. But cause and effect do not occur simultaneously. An economic shock, e.g., an increase in government spending, has a subsequent effect on unemployment, which has a subsequent effect on inflation, which has a subsequent effect on interest rates, etc. These changes constitute the dynamic path by which we move from one equilibrium to another, but simultaneous equation modeling does not show that. The recursive model used in this study is designed to correct for that.

This study is unique in that the parameter estimates in it are generated entirely from empirical estimates of the effect of one variable on another. and the order in which they do it. Empirically we found only one order in which variables' values can be determined, given the estimates of parameters that connect them, that will allow the whole system to be solved, allowing movement from one equilibrium to the next.

In this recursive model, GDP is the result of estimating 11 component equations necessary to pass through all the recursive effects of a shock. Sometimes, several endogenous variables can be solved simultaneously (Section 5 below).

The initial equations solved in our system for determining GDP are solved in the following order:

1. A shock, of a monetary or fiscal nature, causes a change in demand for domestically produced goods. This results in an initial change in GDP.

2. Prior period ratios show how the shock would likely split into production of goods for export and production of goods for domestic demand. The portion produced to satisfy current period domestic demand was taken to directly affect current period unemployment (eq.5.6). It also determines the initial change in the trade deficit, a change derived from empirical evidence of imports being affected in the same period as the change in GDP, but exports only after a two year lag.

   Determinants of unemployment were found to be income, the trade balance, and Inflation. Substituting the determinants of inflation into the unemployment equation shows the initial change in unemployment as a function of initial changes in GDP, the trade deficit and lagged values of unemployment.

3. The inflation rate can then be determined from (now) known changes in GDP, unemployment, and the trade deficit, and lagged values of M1, foreign borrowing and national saving, and some one-time shocks.

4. Then, the prime interest rate can then be determined from these initial changes in GDP, the trade deficit, unemployment and inflation.

5. These four changes allow the calculation of changes in total national saving and its personal, corporate, depreciation and public saving components.

   (To avoid repetition, the order in which the rest of the 25 endogenous variables are solved is given only in Section 5 below.

6. The *initia*l change in GDP caused by the shock generates changes in all the above variables, which themselves are determinants of GDP. These subsequent changes in GDP's determinants due to the initial shock's changes in GDP cause further first period changes in GDP (the feedback loop).

7. This allows for a recalculation of period one GDP which includes all feedback effects.

   For the second period's <u>initial</u> change in GDP, any lagged effects of the first period shock, plus any new shocks, constitute the second period shocks. They cause the initial change in second period GDP. This change in GDP causes changes in unemployment, etc., in period two. These changes permit a final recalculation of second period GDP which includes these feedback effects. The process then repeats itself until shocks stop occurring. Once there are no remaining carryover effects to serve as a next period shock a new equilibrium is reached.

The 2017 econometric model's parameter estimates connecting these variables to each other permit only this one ordering of recursive effects if the new equilibrium is to be determined. It thus avoids the problem of multiple solutions that some models have, identified by Romer (2016).

2. Methodology Used to Construct the Dynamic Path

The recursive IS-LM model developed below is demand driven and micro foundations-based. It bridges the gap between older Keynesian models IS-LM models which are static models. They do not try to estimate the dynamic path in which endogenous variables are solved that connects one equilibrium with another. This newer recursive IS_LM model does that, and brings it in line with more recent New Keynesian and DSGE modeling.

The recursive model's micro-foundations may be explained this way: *money* aggregate demand in the recursive model is determined by the income workers receive, determined by the amount that workers choose to work. The amount they choose to work is driven by their real aggregate demand for goods and services. Work is used to align real and money aggregate demand (to the extent work is available). This recursive model also emphasizes finding the dynamic path that connects one equilibrium to another. In these ways, the new recursive IS-LM model is consistent with modern macroeconomic thinking.

The success of this new recursive model in fitting the data will be compared with that of partial equilibrium IS-LM used in an earlier study Heim 2017). This is done by determining the average difference between estimated and actual values of the two models over the 1981-2010 period for the U.S.

In 2017, this author developed a 56 equation econometric model of the U.S. macroeconomy (Heim 2017). The reviewers included Robert Solow (macroeconomics) and a distinguished econometrician, Kajal Lahiri.. The model has two distinguishing characteristics:

1. The parameters showing how an endogenous variable is connected to its determinants in each partial equilibrium equation are econometrically estimated, and

2. Production in the models is demand driven. Product composition within aggregate demand is driven by standard utility maximization and diminishing returns theory. Profit maximization theory incentivizes the way entrepreneurs choose to produce what is in demand.

The first of these characteristics ensures the models are science-based. The second characteristic stems from many authors' experience that IS-LM demand driven models seem to provide good explanations of how the economy actually works, i.e., fit the actual data better than do other types of models. Even the best DSGE models, seem to be evaluated as inadequate in this regard (Edge and Gurkaynak, 2011, Solow 2017).

The model's 38 behavioral equations, contain estimates of how a change in one explanatory variable in the model affects that equation's dependent variable, holding the other variables in the model constant. Each equation is a partial equilibrium model. Collectively they constitute a general equilibrium model. Each parameter estimate was tested in four time periods between 1960 and 2010, and only results that could be replicated in at least three of the four are used. Tests for identification, stationarity, multicollinearity and heteroscedasticity problems were also made, and standard solutions incorporated as necessary. See the 2017 book for details.

But the 2017 book's individual equations do not deal with the very real problem of feedback loops that augment or diminish the effects of an initial shock on GDP and other variables. This paper attempts to solve that problem by estimating these feedback loops from this same econometric data. It includes the effects of feedback loops in estimates of all policy change effects.

A computer program BASIC256 (2020) was used to create a system for the 1981- 2010 period that would replicate the 2017 model recursively. The recursive model would estimate the values of any endogenous variables used as explanatory variables in the model's endogenous variable equations. The static nature of the partial equilibrium equations allowed it to use actual values for all explanatory variables in estimating parameters. The recursive method requires we uses estimates, not actual values, of any endogenous variables used as explanatory variables. This is the main difference between the models.

The recursive model includes 43 variables, 25 of which are endogenous, or partially endogenous. The initial values for all variables are 1980 data and earlier for variables with lagged effects. The model is straightforward. For comparison purposes, equations and their parameter estimates are identical to those in the 2017 model. The recursive model is estimated, equation by equation, based on the initial effects of a monetary or fiscal shock on GDP. This change in GDP effects the other endogenous variables, unemployment, inflation, interest rates, etc., and are estimated from their structural equations, in the order specified in the introduction and in Section 4 below. All of these variables were found to have feedback effects on the GDP. At the end of the period, a final GDP value for the period was estimated incorporating the feedback effects of these variables. Then model then iteratively repeats the process for the remaining 29 years of forecasting with a simple "For/Next" computer statement. Typically, the new equilibrium was obtained in two - three years, assuming no additional shocks occur.

The difference between the estimated and actual values of variables in each period, shown as a percentage of their actual value for that period, is calculated for both the 2017 model and the recursive model. Results are compared below in Section 5. Percentage differences (PD) were calculated in absolute values to avoid differences adding to zero, essentially for the same reason differences in observations from their means are squared in variance and standard deviation calculations.

$$PD_i = ABS(Estimated_i – Actual\ Value_i) / ABS(Actual\ Value_i)$$

Parameter estimates in the 2017 model initially calculated in using 1st difference models to better control for the biasing effects regression coefficients of multicollinearity and nonstationarity. When converting the findings to levels, typically constants have to be added to represent intercept values for each equation. This was done for the compute model, using constants that minimized the discrepancy between actual and predicted values for variables in the model

Variables and Acronyms Used in the Model

(Column 1)

<u>24 Endogenous & Partly Endogenous</u>

ΔY = Change in GDP

ΔT_T = Change in total taxes

ΔG_T = Change in total government spending

$\Delta Unem$ = Change in % unemployed

$\Delta Infl\%$ = Change in inflation rate

ΔG_{Def} = Change in government spending deficit

ΔLF = Change in loanable funds (total p,c,d&p savings) + foreign borrowing = $S_{P,C,D}$ + For.Bor)

ΔT_{Def} = Change in tax deficit

ΔG_{Def} = Change in govt. spending deficit

ΔM = Change in total imports

ΔX = Change in total exports

ΔC_{Bor} = Change in consumer borrowing

ΔS_{Per} = Change in personal savings (=ΔPerSav)

ΔI = Change in investment

ΔDEP = Change in depreciation (=S_D)

ΔACC = Change in the accelerator ($Y_i – Y_{i-1}$)

ΔSav_{PCD} = Change in gross saving (S_P, S_C S_D &Sp)

ΔSav_P = Change in personal saving

ΔSav_C = Change in corporate saving

ΔSav_D = Change in depreciation saving

Δ (T-G) =Total government receipts – government expenditures (public saving)

C_T =Total Consumer Demand

C_D = Consumer Demand for domestically produced consumer goods & services

(Column 2)

VNam = Dummy variable for Viet Nam military build up years

Reagan, Iraq = Dummy variable for Reagan & Iraq military build up years

'09 Fin.Crisis = Dummy for 2009 financial crisis/recession

OPEC73 = Shock73 = Dummy for OPEC increase in oil prices, 1973

OPEC78 = Dummy for OPEC increase in oil prices, 1978

Shock05 =Katrina Shock05= Dummy for Katrina Hurricane shock

Shock08 = Dummy for 2008 Financial Crisis

BEA DefnChge99 = Dummy for change in unemployment accounting, 1999

Shock08 = Dummy for 2008 Financial Crisis

I_T = V = Total Investment Demand

<u>18 Exogenous Variables</u>

$\Delta M1_{Real}$ = Change in M1 real money supply (As a shock)
ΔCC = Change in Consumer Confidence (Conference Board)
ΔXR_{AV} = Change in average exchange rate (0 to -3)
ΔDJ = Change in NYSE Composite Index
$\Delta WGDP_{RealTP}$ = Change in trade weighted GDP of U.S.'s trading partners
ΔPOP = Change in population growth
ΔPOP_{1665} = Change in population growth of 16-65 workforce
ΔT_T = Change in total taxes (partly)

9 dummy variables to represent one time exogenous events with major economic impact. See right column.

<u>2 Other Definitions</u>

AR(i) = i-th order autocorrelation control
"Period 1" = "Year 1" = Period in which the initial shock occurs

3. LITERATURE REVIEW – EARLIER METHODS OF SOLVING IS-LM MODELS

3.A. Wharton School Econometric Model (Evans & Klein, 1968)

The 76 equation Wharton School model contains 47 endogenously determined equations and 29 identities. Some of the equations are linear, some are nonlinear. The method used to solve systems of nonlinear equations utilizes Taylor series expansions (Newton's method). It is used to simultaneously solve the system for current period (quarter) values (Evans and Klein, 1968, p. 43). Once obtained, assuming no further shocks to the system after those occurring in the initial period (quarter or year), the model is recursively projected forward so that any additional 2nd, 3rd, and 4th period, etc., lagged effects of the shock can be accounted for. This provides the model's dynamic effects. Other variables were held constant while calculating the recursive effects of the shocks.

Evans and Klein present the results of such dynamic simulations in their 1968 book. Dynamic results are presented for 18 key variables behavior over the 8 quarters following an initial shock (p.52). The forecasts are partial equilibrium forecasts from the equation defining that variable's determinants. These forecasts are provided for the various components of consumer and investment spending (9 simulations), the GNP, unemployment, the implicit price deflator, categories of imports, government purchases of goods and services, corporate profits (Evans and Klein, 1968, p.51).

3.B. The Data Resources, Inc. (DRI) Model (Eckstein, 1983)

Eckstein's 1983 800 equation model contains a sub-model of 212 equations that provide a demand driven, i.e., IS-LM explanation of variation in the GDP and its major components over time. Dependent variables for 64 of these equations are stochastically determined. Financial market models and specific industry models account for the rest of the 800 equations. The model's 64 endogenous variable parameters are simultaneously estimated to obtain parameter estimates for each endogenous variable's determinants, i.e., their other endogenous variables, exogenous variables, and lagged variable determinants.

Both *ex ante* and *ex post* methods of showing the model's dynamic results are used. For *ex ante* forecasts, Eckstein calculated future quarter's effects of a current period shock on (e.g.) GDP, based solely on the lagged effects in future years of shock-induced current year changes to the model's explanatory variables. The *ex ante* approach assumes no changes in the underlying exogenous variables occur in forward periods, i.e., no regime change. Eckstein uses such *ex ante* forecasts to compare the DRI model's responses to shocks to a ARIMA model (p.27). Mostly though, he uses the *ex-ante* approach to show that, after a shock, no matter what the effect the shock has on them, the major variables in in his models, e.g., the GDP, unemployment, consumption, etc. return after a few years to a stationary equilibrium; they are not explosive or degenerative. He does this by showing how the multiplier effects of a shock on a specific variable vary each year until the multiplier effect falls to zero (pp. 19, 20).

3.C. Ray Fair's *Estimating How the Macroeconomy Works* Model (2004)

Fair uses a 100 equation model of the U. S. economy (as well as separate models for other countries). The 30 stochastic equations of the U.S. model are nonlinear. The equation system is simultaneously solved using the Gauss-Seidel technique (pp.7 & 14). The model uses *ex-ante* dynamic simulation (pp.14, 193). Root mean square error (RMSE) is used to compare forecasts of future year dynamic effects with what actually occurs. The tests using RMSE are on forecasts beyond the estimation period; to guard against data mining effects on RMSE within the sample period used to estimate the model. (p.15)

3.D. *An Econometric Model of the U.S. Economy* (Heim 2017)

Like the preceding models discussed, this author's 2017 model demand driven model in 56 equations, totally dependent on econometric estimation to provide parameter estimates describing the underlying relationship between variables in the model. Equations describing the determinants of income distribution sides are also estimated. There are 75 separate variables, when we count a variable with one lag used in one equation as a separate variable from the same variable used with a different lag level in another equation. 38 equations are stochastically estimated; 18 are identities

Estimates of differences between projected and actual values of GDP were very similar those found in the earlier simultaneous equations models discussed above.

In addition, an *ex-post* dynamic version of Heim's model was used in the book to compare this Cowles model with a number of DSGE models (including FRB/US) and VAR models (including one of Sims most cited models). In doing these comparisons, models of each kind were estimated using 1960 – 2000 data, and the models were used to forecast GDP for a full ten years (2001-10) beyond the estimation period. RSME was used to measure the goodness of fit. The Heim model outperformed the other models in the out - of - sample test period.

4. The Structural Model Equations

All structural equations in this section are statistically estimated and taken from the econometric model presented in Heim (2017). The data for all variables, with one exception, are yearly data for the 1960 – 2010 period. Virtually all t-statistics for explanatory variables were significant at the 5% level; most at the 1% level Detailed explanations as to variables were selected and their robustness determined are given in Heim (2017). Notationally, periods"-1,-2 and -3"represent the initial conditions data for each period. Period "0" is the initial period a shock (stimulus) is applied,"+1" the period after that, etc.

1. <u>Unemployment: Structural Model</u>

$$\Delta\ \%\ Unem_0 = 1.27 - 0.41\ (\%\Delta\ GDP_0) - .14\ \Delta\ (Infl_0) + .57(Shock73) - .51\ (Shock05) + .75\ (Shock08) \quad R^2 = .84 \quad \text{(Eq. 12.2.2.1.TR)}$$

Substituting inflation's determinants (given in Eq. 11.1.TR) in for the inflation variable and consolidating gives

$$\Delta\ \%\ Unem_0 = 1.50 - 0.485\ \Delta\ (\%\Delta\ GDP_0) + .18\ [\Delta\ (UnemRate_{(-1)} + 22.41\ \Delta\ (\ (M\text{-}X)/Y)_{Real\ AV(0,-1)} - 2.17\ \Delta(ForBor_{-1}/Y_{10})_{Real} + 7.67\Delta(Gross\ Sav_{-1}Y_{-1})_{Real} - .0015\ \Delta\Delta M1_{-1}\ ] + .45\Delta\ (OPEC73\&78\ Shock) - .60\ (shock05) + .89\ (shock08)$$

The computer model for unemployment does not include the three shocks, since they occurred before this study's sample period, 1981-2010. The Lagged average values of values of unemployment in periods (-1,-2) were also found statistically significant upon retesting and included in the computer model. As was the lagged M1 variable. This final model was included in the computer program and is given as

$$\Delta\ \%\ Unem_0 = .18 - 32.94\ \Delta\ (\%\Delta\ GDP_0) + 2.17\ [\Delta\ (UnemRate_{(av\text{-}1\text{-}2)}\ + 54.29\ \Delta\ (\ (M\text{-}X)/Y)_{Real\ AV(0,-1)}\ - 3.95\ \Delta(ForBor_{-1}/Y_{10})_{Real} + 22.68\Delta(Gross\ Sav_{-1}Y_{-1})_{Real}\ + .0046\ \Delta\Delta M1_{-1}$$

Initial changes in exports and imports are predetermined by the initial change in GDP itself, since it is a major determinant of imports, which in turn we find to be a major determinant of exports. See Eq. 5.6 and 6.1 below. Also affecting unemployment the year they occurred were two OPEC oil price increases of the 1970, the hurricane Katrina shock of 2005, and the financial crisis shock of 2008. These are represented by dummy variables

The inflation structural equation (Eq. 11.1.TR) has been substituted into the unemployment structural equation above. This allows the effects of a change in GDP on the unemployment rate due to a shock to be estimated directly from the initial shock to GDP, caused by a change in fiscal or monetary policy. The stimulus-induced initial change in GDP and the initial changes in exports and imports that result from it cause the initial change in unemployment. The recursive process then moves on to solve for inflation, which was not solvable until unemployment rate had been determined. At the end of the recursive process for each period, a final unemployment rate is recalculated, which includes the initial change and any subsequent changes caused by variables solved later in the recursive process. The same process is followed for other variables solved later in the recursive process. Then the change in GDP is recalculated to get a final value of the full change in GDP for the period.

The underlying idea is that a stimulus has an initial (econometrically determined) impact on GDP. This initial GDP change may then recursively affect other variables that are determinants of GDP, and only in the following order. When the initial recursive process is completed, all variables will also be changed. the GDP is then recalculated using the changed variable values. This is the final estimate of GDP for the period.

2. Inflation: Structural Model

$$\Delta(INFL)_0 = -2.20(\Delta Unem_{Av(0\ and\ -1)}) + .009\Delta(\Delta M1_{Real(-2)})\ - 135.67\ \Delta\ (\ (M\text{-}X)/Y)_{Real\ AV(0,-1)} + 13.12\ \Delta(ForBor_{-1}/GDP_{-1})_{Real}\ - 46.46\Delta(Gross\ Sav_{-1}/Y_{-1})_{Real}\ + .30\ \Delta\ Infl_{-1} + .30\ \Delta\ Infl_{-3}\ - .005Unem_{-}\quad R^2 = .78 \qquad (Eq.11.1.TR)$$

To determine inflation, we insert the earlier-determined initial unemployment rate and trade deficit figures , as well as the lagged values of foreign borrowing and national saving found to influence current inflation. Prior year levels of unemployment, as well as the years since NAFTA was in effect and China was in WTO also negatively impacted inflation levels.

In econometric estimation of the inflation equation, the deflationary effect of trade deficits as a percent of GDP was found to be statistically significant and explained 6% of the variation in inflation over the 1960-2010 period.

Lagged Foreign borrowing (positively) , and national saving (negatively) also affected inflation, as did the OPEC shocks of ’73 and ’78 and the persistence of prior inflation levels.

3. Prime Interest Rate: Structural Model

$$PR_0 = .40\ \Delta Infl_0 - 1.12\ \Delta Unem_0\ - 045\ \Delta M1_{-1} + .16\ \Delta M1_{-2.}\ + .000001\ \Delta GDP_{-1} \qquad R^2 = .74 \qquad (M1\ Modified\ Eq.9.2.TR)$$

Once unemployment and inflation are determined the prime interest rate can be estimated. This is a Taylor rule model. Further statistical testing indicated that lagged changes in the money supply was also a factor affecting the Prime rate in the expected ways: a one year lag had a liquidity effect, the two year lag an inflationary lag. M1 is exogenous.

4-7. Total U.S. Saving (Personal, Corporate, Depreciation and Public): Structural Model

Once the unemployment, inflation and Prime rate equations are solved, we can solve the equation for U.S National (total) saving, and each of its component parts:

Total Savings = Personal + Corporate + Depreciation +Public Savings = $S_{PCD(0)} = S_{P(0)} + S_{C(0)} + S_{D(0)} + S_{Pub}$

$$\Delta S_{P(0)} = -17.89 + .22\Delta(Y_0-T_{T(0)})^{1.3} -.14\Delta T_{DEF(0)} +.14\Delta(PR^3) -2.27\text{ E-25 } \Delta DJAV_{(0,-2)}{}^9 -725.37\Delta(ICC_{-1})^{0.1} -.16\ \Delta CBOR_{(0)} -195.94(\text{BEA DefChge 99}) -716.30(\text{Katrina Shock 05}) -.03\Delta(INFL_0{}^3) \quad R^2=.79 \quad \text{(Eq. 13.3.1.TR)}$$

$$\Delta S_{C(0)} = .66\Delta T_{DEF(0)} -.77\Delta G_{DEF(0)} -.17\Delta ACC_0 \quad R^2=.64 \quad \text{(Eq. 13.1.2.TR)}$$

$$\Delta S_{D(0)} = .06\ \Delta I_0 +.10\ \Delta I_{-1} +.10\ \Delta I_{-2} +.07\ \Delta I_{-3} +.03\ \Delta I_{-4} +.04\ \Delta I_{-5} +.03\ \Delta I_{-6to-10} +.04\ \Delta I_{-11to-17} +.13\ AR(8) \quad R^2=.97 \quad \text{(Eq. 13.2.1.TR)}$$

$$\Delta S_{Pub(0)} = (T_{Total-} - \text{Transfer Payments}_{(0))}) - \Delta G_{GDP(0)} -.$$

Total National Saving = Eqs. 13.3.1TR(R^2=.79) + 13.1.2TR(R^2=.64) + 13.2.1TR(R^2=.97) + (T[i]-TP[i])-G[i], or

$\Delta S_{pcdp(0)}$ = -17.89 +.22*(Y[i]-T_T[i]) -.14*(T_T[i]-TRANSPAY[i]) + .14*(PR[i]^3) -725.37*(CC1[i-1]^.1-(CC1[i-2])^.1) -.03*(INFL^3) -.16*T_T[i]- 2.29E-25*(DJ[i]^9 +DJ[i-2]^9) +.66*(T_T[i]-TP[i]) -.77*G[i] -.17*(Y[i]-Y[i-1]) + .06*(V1[i]-V1[i-1]) + .10*(V1[i-1]-V1[i-2])+ .10*(V1[i-2]-V1[i-3])+.07*(V1[i-3]-V1[i-4]) +.03*(V1[i-4]-V1[i-5])+.04*(V1[i-5]-V1[i-6]) +0.03*(V1[i-6]+V1[i-7]+V1[i-8]+V1[i-9]+V1[i-10])-0.03*(V1[i-7]+V1[i-8]+V1[i-9]+V1[i-10]+V1[i-11]) +0.04*(V1[i-12]+V1[i-13]+V1[i-14]+V1[i-15]+V1[i-16] +V1[i-17]+V1[i-18]) -0.04*(V1[i-11]+V1[i-12]+V1[i-13]+V1[i-14]+V1[i-15]+V1[i-16]+V1[i-17]) +(T[i]-TP[i])-G[i]

Which after consolidation and substitution of investment's determinants in for V1[i], permits solving for Total National Savings and its four component parts are determined as a function of these pre-determined variables, some lagged values of investment some exogenous variables, and the initial change in change in GDP. The exogenous variables are POP_{21-31}, FB, DJAV, and ICC.

8- Total Taxes (T_T): Structural Model

Total taxes can also be determined after the shock to GDP, unemployment and inflation are determined

$$\Delta T_{Tot(0)} = -11.17 +.30\Delta Y_0 +13.64\Delta\ Infl_{AV-1-2} - 50.87\Delta Unem_0 + .43\ AR(1) \quad R^2=.72 \quad \text{(Eq.14.1.TR)}$$

AR(1) is included to control serial correlation.

9. Total Government Spending ($G_{T\&I}$): Structural Model

$$\Delta G_{T\&I(0)} = 61.69 +.03\Delta Y_0 +23.85\Delta Unem_0 + .028\Delta Pop_{-21-31} + 64.65(\text{VNam}) + 34.85\ (\text{Reagan, Iraq}) +74.56(\text{'08Fin.Crisis}) \quad R^2=.66 \quad \text{(Eq.15.1.TR)}$$

Pop_{-21-31} is not included in the computer model because it is treated as lagged 21 years, hence cannot be affected by current period shocks, and therefore can't cause changes in current period government spending due to the shock.

10. Tax Deficit (T_{Def}): Structural Model

$$\Delta T_{Def(0)} = (\Delta T_{Tot(0)} - \Delta TP_{0()}) = -11.17 +.30\Delta Y_0 +13.64\ \Delta Infl_{AV-1-2} - 50.87\Delta Unem_0 +.43(AR_1) - \Delta TP_0 \quad R^2=.72 \quad \text{(Eq.14.1.TR-TP)}$$

This is the same model as for Total Taxes (T_T), except for subtractions of transfer payments (TF). This is to compensate for the fact that in computing the deficit we use only the government expenditures included in the GDP definition of "G", which excludes spending on transfers.

11. Government Spending Deficits (G_{Def})

$$G_{Def(0)} = (G_{T\&I(0)} - LF_0) = 61.69 + .03Y_0 +23.85UNEM_0 + .028POP_{-21-31} + 64.65(\text{VNam}) + 34.85\ (\text{Reagan, Iraq}) +74.56(\text{'09Fin.Crisis} \quad R^2=.66 \quad \text{(Eq.15.1.1TR-TP)}$$

This is the same model as for Total Government Spending ($G_{T\&I}$), except for subtraction of transfer payments.

12. Imports: Structural Model

With taxes and government spending level equations now solved, we can determine the level of imports and exports.

Total Imports = $M_{T(1)} = M_{C(1)} + M_{I(1)}$

$\Delta M_{C(0)} = .19\Delta(Y_0 - T_{Tot(0)}) + .25\Delta T_{DEF(0)} - .186\Delta G_{DEF(0)} - 3.06\Delta PR_0 + 2.22\Delta XR_{AV(0)}$ **R^2=.77** **(Eq 4.2.TR)**

$\Delta M_{I(0)} = .045\Delta T_{DEF(0)} - 3.91\Delta PR_{-2} + .62\Delta DJAV_{(0)}$ **R^2=.72** **(Eq. 5.6.TR)**

Consolidating to get total imports gives

$\Delta M = (\Delta M_{Con(0)} + \Delta M_{Inv(0)}) = \Delta M_1 = .19\Delta(Y_0 - T_{Tot(0)}) + .295\Delta T_{DEF(0)} - .186\Delta G_{DEF(01)} - 3.06\Delta PR_0 - 3.91\Delta PR_{-2} + 2.22\Delta XR_0$

13. Exports Model

$\Delta X = .16\ \Delta(WGDP_{RealTP(-2)}) - 9.47\ \Delta(XR_{AV0\ to\ -3}) + .56\ \Delta(M) + 14.74\ \Delta(PR_{RealAV\ -1\ -2}) - 11.58\ \Delta INFL_{AV-1\ to\ -2}) - .49\ AR(6)$ **R^2=.88** **(Eq.6.1.TR)**

14. Consumer Borrowing Model

$C_{Bor(0)} = .44Y_0 - .44T_{Tot(0)} + .47\ T_{Def(0)} - .46\ G_{Def(0)} - 14.62PR_0 - 1.37DJ_{-1} + 19.84XRAV_{(0)} - .07(\Sigma S_{P\ AV0-9} + \Sigma(M2-M1)_{0-3}$ **R^2=.52** **(Eq.4.6.TR)**

15. Total Investment (I_T) Model

$\Delta I_{T(0)} = .25\Delta ACC_0 + .30\Delta TDEF_{(0)} - .32\Delta G_{DEF(0)} + .97\Delta DEP_0 - 10.53\Delta PR_{-2} + .87\Delta DJAV_{(0)} + 3.18\Delta XR_{AV(0)}$ **R^2=.87** **(Eq.5.2.TR)**

Substituting the definition of depreciation (DEP = S_D), which is:

$\Delta S_D = .06\ \Delta I_0 + .10\ \Delta I_{-1} + .10\ \Delta I_{-2} + .07\ \Delta I_{-3} + .03\ \Delta I_{-4} + .04\ \Delta I_{-54} + .03\ \Delta I_{-65to-10} + .04\ \Delta I_{-111to-17}$ **R^2=.97** **(Eq.13.2.1.TR)**

into the investment equation and consolidating the current period (period 0) Investment variables on the left hand side of the equation, we get

$\Delta I_T = .265\Delta ACC_0 + .32\Delta T_{DEF(0)} - .34\Delta G_{DEF(0)}\ + .11\ \Delta I_{-1}\ + .11\ \Delta I_{-2} + .07\ \Delta I_{-3} + .03\ \Delta I_{-4} + .04\ \Delta I_{-5} + .03\ \Delta I_{-6to-10}$
$+ .04\ \Delta I_{-110to-17}\ - 11.19\Delta PR_{-2} + .92\Delta DJ_{AV(0)} + 3.38\Delta XR_{AV(0)}$

16. . The Total Consumption Model

$\Delta C_T = .54\Delta(Y - T_T) + .55\Delta(T_T) - 12.21\Delta(G_{T\&I}) - 9.31\Delta PR + .48\ \Delta DJ_{-2} + .010\Delta POP + .41\Delta ICC_{-1} + 36.80\Delta M2_{AV}$
(t =) (10.8) (11.0) (-7.9) (-4.6) (5.4) (4.3) (1.2) (4.3)

$+ .14\ \Delta C_{B2}\ - 418.25PPOP_{16}$ R^2=94.8%
(Eq. 4.1T.TR modified to include age distribution variable POP_{16})
(3.6)

17. M1 Effects on C and I Models

Econometric estimates of the determinants of residential investment indicate a current period increase in new housing when the M1 money supply is increased. Specifically, the estimated effect is ΔI_{Res} = $.37\Delta M1_{Real}$ using CPI deflator to deflate the nominal M1 money supply. Using the GDP implicit price deflator, which we did, the effect was found to be $\Delta I_{Res} = .21\Delta M1_{Rea)}$ .. Testing also indicated that the same M1 change, two years later, had an impact on real consumer services demand of $\Delta C_{S(2)}$ = $+.19\Delta M1_{Real}$ using the implicit price deflator.

Despite the strong positive current period effect of M1 on the housing market ($I_{Res(0)}$), which is typically about a third of total investment, a change in M1 was not found to have a statistically significant impact on investment ($I_{T(0)}$) in total. The cause appears to be the relatively strong simple correlation (-.36) between investment in fixed plant and equipment, which is roughly 2/3 of total investment, and M1, offsetting M1's positive effect on residential M1.

Our standard way for calculating the initial effects on GDP of a shock is to use the IS curve method. The IS curve method involves adding together the components of GDP. This can be done either of two ways:

$$GDP = C_D + I_D + G_D + X_D, \text{ or: } C_T + I_T + G_T + (X_T - M_T)$$

More precisely, adding together the determinants of these components of GDP multiplied by their parameter estimates. GDP estimates obtained this way in Heim (2017) were remarkably successful in matching actual GDP variation, varying only by about 0.50% on average. (See Cptrs 8 and 16).

Econometrically, parameters for the determinants of C,I,G,X and M are estimated *post hoc*, using actual data for each determinant and adding together parameter estimates into one equation obtained from estimating the consumption function itself, with the estimates of I,G,X and M's determinants obtained from testing these functions separately.

5. Findings

5.1 Recursive and Partial Equilibrium Methods

Historically, large scale IS-LM model parameters were estimated simultaneously, (Evens and Klein 1968, Eckstein 1983 and Fair 2004). The parameter estimates obtained were then used to recursively examine the effects of a shock, usually changes jn government tax, spending or money supply.

The Heim 2017 model was calculated differently. Partial equilibrium models for each endogenous variable were estimated and summed into one IS equation to estimate GDP. Parameter estimates were estimated separately for each endogenous variable, from that variable's own determinants. Actual data was used for all right-hand side variables, both endogenous and exogenous when estimating endogenous variable equations. This was done to maximize the fit of the equation for some particular endogenous variable to its determinants, using east squares or two-stage least squares as appropriate and 1960-2000 data. Then those parameter estimates explained the actual data for the out of-sample ten years (2001-10) that followed. Results suggested the model worked well. The average difference between actual and predicted values were 0.5% (GDP and consumption), 1.5% (exports) and 3.4% (investment). Differences were expressed as a percent off the variables actual value. (Heim 2017, Table 16.1.1).

This paper compares that 2017mehodology with the recursive methodology using the same parameter estimates from the same structural equations used in the 2017 model. Hence, the testing is done here simply determines whether recursively obtained *estimated* values for endogenous variables explain the data better than models that use only *actual* values of explanatory variables, even endogenous ones. Estimating an endogenous variable's value for any period from the estimated, rather than actual values of other endogenous variables in its equation typically will yield different results. *A priori*, one would expect estimates of a left-hand side variable based only on the actual values of its determinants would be more accurate than estimates that are based on a mix of actual values for exogenous variables and estimated values for endogenous variables.

5.2. Empirical Findings

Our first test will be to see how accurately the recursive model performed when the actual real changes in government spending, receipts and money are used as the shocks in each period, All the endogenous variables effects are estimated recursively in response to the shock. Using the recursive method, results for 1981-2010 indicate the average yearly difference between estimated and actual values of GDP was 5.9%of GDP. When using the 2017 method, this difference was only was only about 2/10 of 1%. Results are shown In Table 1.1 below.

Table 5.1
GDP Dynamic Effects Estimated GDP
Compared to Actual

	Year	GDP Calculated Using Recursively Obtained *Estimates* Of Endogen-ous Variables Values Used to Estimate GDP	GDP Calculated Using *Actual* Values of Endogenous Variables Used to Estimate GDP	Actual GDP
Pre – Stimulus	(1980)	$ 5838	$ 5838	$ 5838
Model Estimate	(1981):	$ 5871	$ 5939	$ 5987
	(1990):	$ 8085	$ 7985	$ 8033
	(2000):	$ 10159	$11232	$11228
	(2010):	$ 12258	$13376	$13249
Average Deviation From Actual, as a % of GDP(1981-2910)		5.9%	0.2%	

Source: Computer Model: Dynamics 4,Heim (2017) Econometric Model Structural Equations, , ERP 2013: Actual Real GDP, Chain deflator 2005=100

The results suggest that, despite its theoretical attractiveness from the recursive model are not likely to be as accurate as estimates derived partial equilibrium models. Since the parameters used to connect one variable to another were the same in both models, attribute this to the fact some of the determinants' "data" in recursive models are not actual values but estimates. Hence, the unemployment estimate in the recursive model's inflation equation is a previous estimate, not actual value, leading to a less accurate inflation estimate. Similarly, both the unemployment and inflation "data" are estimates, not actual data in the equation used to estimate interest rates.

Average estimation errors for 15 the model's key endogenous variables are shown in Table 1.2 below.

Estimation errors for 12 of the 15 variables were noticeably smaller when estimated using the partial equilibrium method than those obtained using the recursive method.

Table 5.2

Average Estimation Error For 1981-2010 as a Multiple of Actual Value
Of Actual T, G and M Stimulus Programs During This Period

Endogenous Variable	Calculated Using Recursively Obtained *Estimates* Of Endogen-ous Variables Values Used to Estimate GDP	Calculated Using *Actual* Values of Endogenous Variables When Estimating GDP	Exog.Var Where Actual GDP= Estimatd
GDP (X only method)	6.0%	0.2%	
GDP (X-M method)	6.2%	0.3%	
Total Consumption	23.5%	1.1%	
Total Investment	24.0%	9.3%	

Imports	29.0%	32.6%	
Exports	67%	12.8%	
Trade Balance (X-M)	116.0%	408%	
Unemployment	4.1%	7.0%	
Inflation	96.0%	53.9%	
Prime Interest Rate	77.0%	75.8%	
Consumer Borrowing	61%	29.6%	
Total Saving	44.0%	4.7%	
Personal Saving	23.0%	19.1%	
Corporate Saving	68.0%	33.1%	
Depreciation Saving	22.0%	10.5%	
Public Saving	0	0,0%	Always Exog.
Govt Spending (GDP Defin.)	0.0%	0.0%	Always Exog.
Total Govt. Spending	0.0	0.0%	Always Exog.
Total Govt Receipts	0.0	0.0%	Always Exog.
M1	0.0%	0.0%	Always Exog.

Finally, though the recursive model may less accurate than traditional IS-LM methods, we need to determine if its policy conclusions are any different than those obtained from more traditional methods. Table 1.3 below suggests that in general the answer is no. Seven different fiscal and monetary policies whose results using more traditional methods are Keynesian are shown below. They all involve changing current levels of government spending, taxes or the money supply by $1.6 billion, roughly the amount of of change that occurred in 2019-2021 in response to the Covid supply chain crises. In some cases we look at both additions and subtractions of that amount.

At the policy implications level, the results prove to be prove to be nearly identical to though obtained using the partial equilibrium of solving the each of the equations for GDP's explanatory variables separately, and then arithmetically summing parameter estimates into one IS curve, which has the LM curve already substituted into it. See (Heim 2017 Eq. 8.1.2.1TR for fiscal policy effects; 4.11, 4.12TR, 5.11TR for monetary effects). These results indicate that,

1. because of "crowd out", the net effect of increasing government spending or decreasing taxes is the opposite of standard Keynesian predictions, but the result is pure Keynesian if the tax cut or spending increase is accompanied by an increase in the money supply.

2. accommodating changes in the money supply can restore the stimulus effect of tax cut, but must be larger than the spending increase in order for spending increases to have a stimulus effect.

3. increases in the money supply alone can also substantially stimulate the economy.

4. Because of crowd out, increasing taxes can be stimulative. The increase in taxes increases loanable funds available for investment by the same amount, but only reduces loanable funds previously available by a fraction of that given by the marginal propensity to save. since tax increases are typically financed out of both reduced spending and saving.

5. Of all the policies, Simultaneously increasing the money supply and taxes seems to provide the most stimulus.

6. The balanced budget multiplier is negative. The decline in GDP due to adding $1600 to government spending is greater than an increase due to raising taxes. . Again, this is because of the greater decline in GDP caused by unaccommodated increases in government spending, than the increase in GDP associated with the tax increase.

Table 5.3

Simulated Effects On GDP Of Adding Stimulus
To Actual Values of T, G and M1

Additional Stimulus Policy	(Year)	Recursive Model GDP Results Obtained Using *Actual* Tax, - Spending and Money Stimulus that Occurred	GDP Results Obtained Using *Additional* Tax, Spending and Money Stimulus	Change in Real GDP .
+0	(2010)	$ 12258	$ 12258	$ 0
+1600 G Spend	(2010):	$ 12258	$ 10767	$ -1491
+1600 GSp+M1	(2010):	$ 12258	$ 12200	$ - 58
+1600 Taxes	(2010):	$ 12258	$ 13269	$+1011
+1600 M1	(2010):	$ 12258	$ 13976	$+1698
+1600 M1+Tax	(2010):	$ 12258	$ 14983	$+2725
+1600 GSp+Tax	(2010):	$ 12258	$ 11783	$ - 475

Source: Computer Model: Dynamics 4,Heim (2017) Econometric Model Structural Equations, , ERP 2013: Actual Real GDP, Chain deflator 2005=100

7. Conclusions

The recursive, or dynamic, IS-LM model follows the pattern of recent dynamic New Keynesian models. It is intended to narrow the gap between the two models.

Many economists will find the recursive model used here to be more theoretically appealing than comparative statics models used in earlier IS-LM studies. This is because the explicit cause and effect relationships in the recursive model show the dynamic path that moves an economy from one equilibrium to another. That said, he recursive model also leads to the same policy conclusions, as the 2017 partial equilibrium method, a static model.

One deficiency of the recursive model is period-by-period the estimated values of endogenous economic variables is not as accurate as the partial equilibrium model, which uses actual dta for all right-hand side variables in each equation, including endogenous. Future improvements in the explanatory power of the underlying endogenous equations in the model should improve the fit of recursive models, because recursively obtained estimates will become closer to actual values. This will bring estimates of endogenous explanatory variables them closer to the actual values used in the old static IS-LM models but at best only match them, since 100% explanatory power of the equations in the recursive model would mean only actual data were used on the right side of theequations in he recursive model.

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